A simplified model for evaluation of fatigue damage in frames

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ABSTRACT: Many studies have been developed to analyze the structural seismic behavior through the damage index concept. The evaluation of this index has been employed to quantify the safety of new and existing structures and, also, to establish a framework for seismic retrofitting decision making of structures. Most proposed models are based in a postearthquake evaluation in such a way they uncouple the structural response from the damage evaluation. In this paper, a generalization of the model by Flórez-López (1995) is proposed. The formulation employs irreversible thermodynamics and internal state variable theory applied to the study of beams and frames and it allows an explicit coupling between the degradation and the structural mechanical behavior. A damage index is defined in order to model elastoplasticity coupled with damage and fatigue damage.

1 INTRODUCTION

Damage indices aim to provide a means of quantify numerically the degradation in structures sustained under earthquake loading and therefore a structural safety measure. Their evaluation is very important for seismic retrofitting processes and to define the safety of new and existing structures.

Reviewing the literature existing about the subject, it can be infered that many different models have been proposed through the years to evaluate the structural damage (Bannon et al. 1981, Di Pasquale et al. 1990, Park & Ang 1985). Because of it, the characterization of the damage can be considered a subjective matter so the main problem is its quantification. Most of proposed models do not include explicitly the effect of damage on the structural behavior. This type of calculations neglects the coupling between the damage and the strains which simplifies the analysis but it is not consistent with the physical reality.

For a good accuracy the coupling between damage and strain must be considered all over the structure. Although this latter consideration makes the calculation more complex, the models are more consistent with the definition of damage as a phenomenon with mechanical consequences. Such models are formulated through the principles of Continuum Damage Mechanics. According to these, damage becomes an internal variable of the constitutive relations and a state variable of the problem. Damage index is introduced using the notion of effective stress and the hypotheses of strain equivalence.

However, because of its complexity, continuum mechanics is not the most suitable tool for the analysis of many structures. In order to simplify the continuum problem, a model adapted to frame analysis has been proposed by Cipollina et al. (1995). The proposed formulation is based in a generalization of the lumped plasticity models which allows to include the damage effects. It can be considered as a simplified damage mechanics that incorporates concepts of Continuum Damage Mecha- nics.

In the previous simplified model the damage is a function only of peak values of response variables, therefore the computation of damage neglects any path dependence and cyclic loading effects can not be represented. It is the purpose of this paper to present a formulation to take into account the degradation produced not only by peak values but also by cumulative effects. The proposed approach can handle in an unified way monotonic loading and low cycle fatigue.

In Section 2, a brief description of the simplified model is reviewed. The thermodynamic framework of the proposed damage model is developed in Section 3. From this framework, a damage mechanism is characterized in the following section to describe the fatigue damage. Experimental validations are given to illustrate the applicability of the model.

2 LUMPED DISSIPATION MODEL

Using the same approach that Flórez-López (1995), we consider a formulation that generalize the lumped plasticity models to include the damage effects. According to the lumped plasticity models a frame member is idealized by considering that the element is elastic and by lumping the plastic deformations (rotations and axial elongations) at its ends (Fig.1). This model is consistent with the traditional assumption of plastic hinge (only plastic rotation), bar hinge (only axial deformation) and generalized plastic hinge (combined axial force and bending moment).

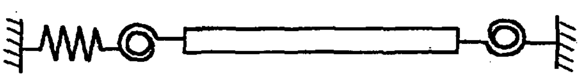


Figure 1. Mechanical model

The stress distribution for each element is described by a three component vector, q, that collects the bending moments at the two ends and the axial force. The corresponding kinematic variables, u, define the deformed shape of the element excluding the rigid body motion (Fig.2):

$$q^t = \begin{pmatrix} M_i & M_j & N \end{pmatrix}$$
$$u^t = \begin{pmatrix} \theta_i & \theta_j & \delta \end{pmatrix} \tag{1}$$

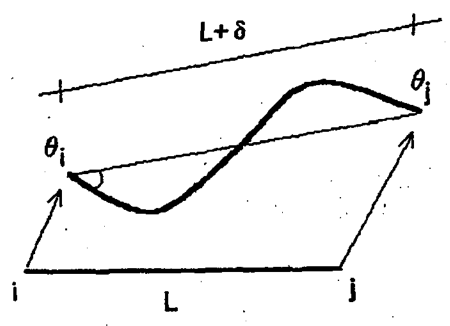


Figure 2. Generalized deformations for the model

To include the damage effects, we suppose that all the dissipative phenomena are concentrated at the hinges. In this way, the plastic deformations and the structural damage are measured through a set of parameters which are defined in the hinges. We will call to this representation as lumped dissipation model.

2.1 *Constitutive equations*

The constitutive equations expressing the relations between the generalized stresses and the deformations is obtained combining the lumped dissipation models with the concepts of Continuum Damage Mechanics.

Performing this for the particular case of a truss member, we obtain the following relationship between the axial force and the damage elongation, δ^d (Cipollina et al. 1995):

$$\delta^d = \frac{d_a}{1-d_a}\frac{NL}{EA} \tag{2}$$

For flexural effects, a similar relation is supposed by postulating the existence of a set of damage parameters D^t=(d_i d_j d_a), taking values between 0 and 1, defined over the hinges:

$$\begin{aligned} \theta_i^d &= \frac{d_i}{1-d_i}\frac{L}{4EI}M_i \\ \theta_j^d &= \frac{d_j}{1-dj}\frac{L}{4EI}M_j \end{aligned} \tag{3}$$

According to this, the constitutive law is therefore given by:

$$u-u^p = u^b + u^d = \left[F^o + F^d\right]q = F(D)q \tag{4}$$

where u^b, u^p and u^d represent the elastic, plastic and damage deformations, respectively, and F^o and F^d the corresponding elastic and damage flexibility matrices.

2.2 *Hysteretic effects*

To describe the behavior of the model under cyclic and seismic loading we suppose an unilateral behavior. According to this assumption, the damage originated by positive actions has no influence on the behavior in compression and viceversa.

We extend the monotonic model by defining two sets of damage parameters, one for positive actions, D^+, and the other for negative actions, D^-. The constitutive equations are given by:

$$u-u^p = F\left(D^+\right)\langle q\rangle_+ + F\left(D^-\right)\langle q\rangle_- \tag{5}$$

3 THERMODYNAMIC BASIS

In order to introduce both damage and plastic processes, we consider a free energy potential of the form:

$$\psi = U^e\left(u, u^p, D\right) + U^p\left(\beta^p, \beta^d\right) \tag{6}$$

where U^e is the elastic strain energy which can be defined as:

$$U^e(u,D) = \frac{1}{2}\left(u-u^p\right)^t K(D)\left(u-u^p\right) \tag{7}$$

where

$$K(D) = [F(D)]^{-1} \tag{8}$$

In addition, U^p denotes a plastic-damage potential, function of the hardening terms, β^p y β^d.

Confining our attention to the purely mechanical theory, the Clausius-Duhem inequality takes the form:

$$-d\psi + q du \geq 0 \tag{9}$$

By derivation of ψ we obtain the dissipative inequalities

$$\begin{aligned} &-\frac{\partial \psi}{\partial D} dD - \frac{\partial \psi}{\partial \beta^d} d\beta^d \geq 0 \\ &-\frac{\partial \psi}{\partial \beta^p} d\beta^p + q du^p \geq 0 \end{aligned} \tag{10}$$

The term

$$Y = -\frac{\partial \psi}{\partial D} \tag{11}$$

represents the thermodynamic force (damage energy release rate) conjugate to the damage variable in the sense defined by Lemaitre (1985) and its value is always positive.

3.1 *Evolution laws*

To formulate the flow rules for plasticity and damage, it is necessary to define two dissipation potentials, φ_p y φ_d, for both phenomena.

The Principle of Maximum Plastic Dissipation implies the normality of the flow rules in generali-zed stress space for plastic deformations and in Y-space for damage variables:

$$du^p = d\lambda^p \frac{\partial \varphi_p}{\partial q} \qquad dD = d\lambda^d \frac{\partial \varphi_d}{\partial Y} \tag{12}$$

where $d\lambda^p$ and $d\lambda^d$ are plastic and damage consistency parameters, respectively.

In the associative case, the dissipation potentials coincide with the plastic and damage functions. The expression for these two functions is obtained from experimental results and their consideration will be treated in the next section.

4 DISSIPATIVE POTENTIALS FOR REINFORCED CONCRETE MEMBERS

The choice of the dissipative potentials is the critical point of the theory presented previously. These potentials should allow modeling those phenomena involved in the problem subjected to study such as fatigue, damage by tension, buckling, etc. Usually, damage is associated only with the elastic strains through the damage energy release rate (Lemaitre 1985). This treatment amounts to that damage is a function on the maximum amplitude of cyclic deformation experienced by the member but does not depend on the cumulative values. Phenomena such as low cycle fatigue, which are very often presented under seismic actions, are uncoupled to damage. To avoid this problem, Ju (1989) proposed a redefinition of the damage energy release rate. However, using the same definition of the energy release rate, it is possible to take into account the fatigue through a suitable definition of the dissipative potentials.

Degradation of material due to fatigue is a quite complex problem as it can be checked in the existing bibliography (Dubè et al. 1996, Marigo 1985, Suaris et al. 1990). Most of models to quantify fatigue damage use extrapolations for the Miner rule (1945), i.e.

$$D = \sum_i \frac{n_i}{N_f} \tag{13}$$

where n_i is the number of cycles for the current amplitude and N_f is the number of cycles to failure for this amplitude. This hypothesis supposes a linear accumulation of the damage and the history of the previous cycles is not taken into account. Moreover, the number of cycles has a very clear meaning in case of harmonic loads, but when random or non harmonic loads, as seismic action, are considered the concept of cycle misses its meaning.

The number of cycles to failure can be calculated by the well known Manson-Coffin criterion:

$$\Delta\varepsilon_p = C(N_f)^K \tag{14}$$

where C and K are material constants and $\Delta\varepsilon_p$ is the width of the plastic hysteresis loop of the cycle. A similar expression can be found using total strain instead of plastic strain (Fig. 3). An experimental fit to this relation was obtained by Mander et al.(1994) for reinforced concrete:

$$\Delta\varepsilon_t = 0.08(N_f)^{-0.33} \tag{15}$$

which can be used directly to define the number of cycles to failure for a given deformation $\Delta\varepsilon_t$.

This expression considers that the failure is mainly controlled by the flexural behaviour of the member. Another expression was proposed by Kunnath et al (1996) considering also the effects of axial and shear forces.

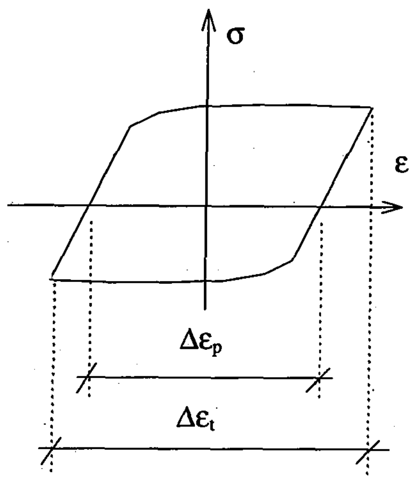


Figure 3. Total and plastic strain amplitude

From the dissipative potentials used in Flórez-López (1995), the following functions are proposed here in order to simulate the low cycle fatigue:

$$g = Y - [Y_{cr} + Z(D)]\xi(\omega) \tag{16}$$

where ω is a cumulative parameter that will be defined afterwards. The function, $\xi(\omega)$, is required to satisfy the following conditions

$$\begin{aligned} \xi(\omega) = 1 &\Leftrightarrow \omega \le \omega_{min} \\ \xi(\omega) = 0 &\Leftrightarrow \omega = \omega_{max} \end{aligned} \tag{17}$$

Figure 4 represents the shape of the new damage function. This model is equivalent to introduce a term of isotropic softening as sometimes is done in some plasticity models.

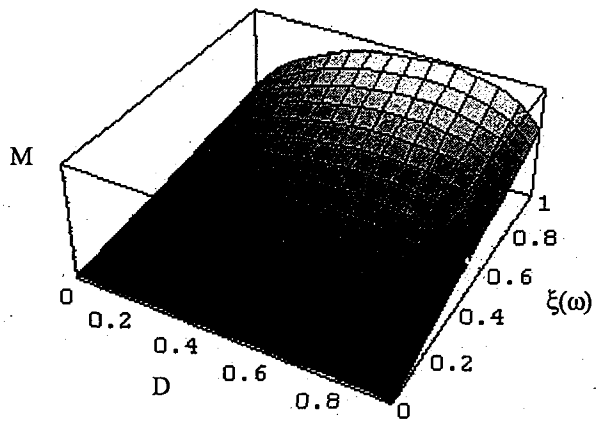


Figure 4. Surface g=0 in the Moment-Damage-ξ space

In the same way, the coupling between damage and plasticity requires to introduce some modifications in the plastic function. The new function is written as

$$f = \left|M - X\xi(\omega)\right| - \left(M_y + R\right) \tag{18}$$

Some different evaluations have been done in order to determine the shape of the $\xi(\omega)$. Good correlation between experimental and analytical results have been obtained with a function such as

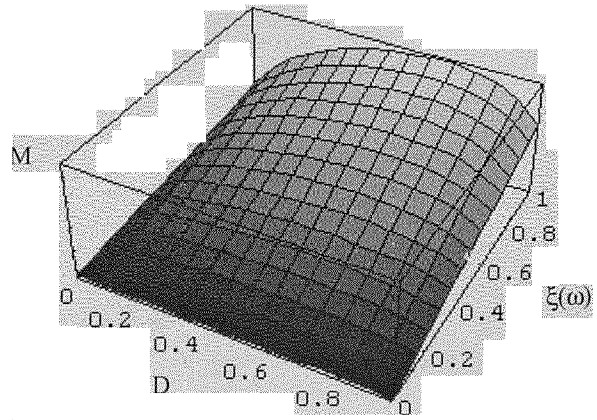
$$\xi\left(\tilde{\theta},\theta_t\right) = 1 - \left(\frac{\tilde{\theta}}{N_f(\theta_t)\theta_t}\right)^{\frac{k}{\mu}} \tag{19}$$

where $\tilde{\theta}$ is the total cumulative rotation, θ_t is the total rotation, μ is the ductility, $N_f(\theta_t)$ is the number of cycles to failure (total rotation function) and k is a constant dependent on the member geometry. The number of cycles to failure can be expressed according to Equation 15. Previously a modification of this equation is required in order to transform total strains in total rotations. A plastic hinge length must be defined. The relationship proposed by Park and Paulay (1975) has been used here.

Quantifying the number of cycles by mean of the total cumulative rotation allows modeling arbitrary loading histories without considering the concept of cycle.

5 EXAMPLES

With the model previously presented some examples have been computed. Figure 5 shows the experimental (Chai et al. 1991) and numerical results of the hysteretic response of a circular cross section column of reinforced concrete retrofitted with a steel jacket. The column was subjected to a constant axial load of 400 kips and lateral displacement was piloted. The parameters of the model has been computed using the following characteristics: EI/L = 2498 kips·in, $M_{cr}^{+} = M_{cr}^{-} = 2160$ kips/in, $M_p^{+} = M_p^{-} = 5940$ kips/in, $M_u^{+} = M_u^{-} = 9300$ kips/in, $\theta_{pu}^{+} = \theta_{pu}^{-} = 0.03$, $\alpha^{+} = \alpha^{-} = 0.7$. Dissipative functions do not introduce the new term $\xi(\omega)$ so fatigue degradation can not be modeled.



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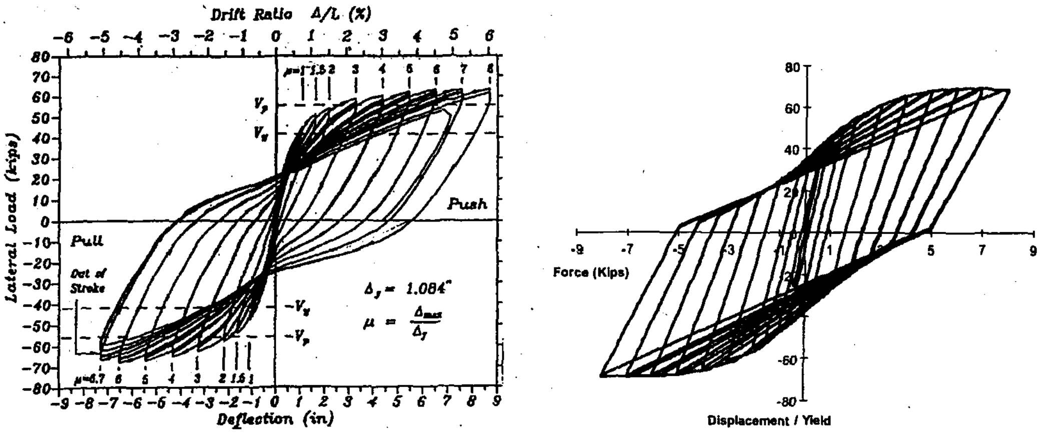

Figure 5. Experimental test (left) by Chai et al. (1991) and numerical simulation (right)

Figures 6 and 7 represent experimental and numerical results using the new dissipative function. Results from Figure 6 are refered to a circular cross section reinforced concrete column (Kunnath et al., 1997) which is subjected to a constant axial load of 806 kN and lateral displacement are piloted. The numerical simulation has be done with the followed parameters: EI/L = 2.51E+7 Nm, $M_{cr}^{+} = M_{cr}^{-} =$ 27.420 kNm, $M_p^{+} = M_p^{-} =$ 87.808 kNm, $M_u^{+} = M_u^{-} =$ 98.784 kNm, $\theta_{pu}^{+} = \theta_{pu}^{-} = 0.029$, $\alpha^{+} = \alpha^{-} = 1$ and k=1.5

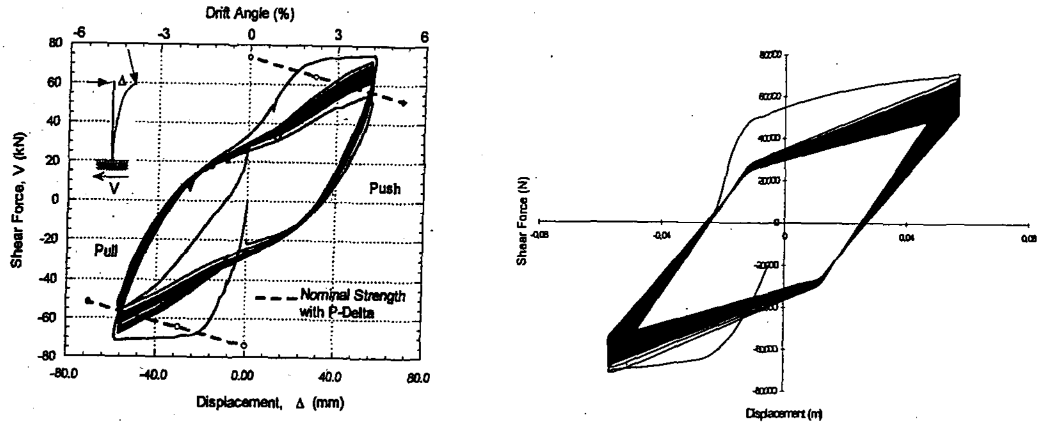

Figure 6. Experimental test (left) by Kunnath et al (1997) and numerical simulation (right)

Figure 7 shows experimental and numerical results of a rectangular cross section reinforced concrete column with moderate confinement tested by Wehbe et al. (1996). As in the previous cases the column is subjected to a constant axial load of 641 kN and lateral displacement are piloted. The numerical simulation has be done using the followed parameters: EI/L = 2.21E+7 Nm, $M_{cr}^{+} = M_{cr}^{-} =$ 210 kNm, $M_p^{+} = M_p^{-} =$ 643 kNm, $M_u^{+} = M_u^{-} =$ 850 kNm, $\theta_{pu}^{+} = \theta_{pu}^{-} = 0.05$, $\alpha^{+} = \alpha^{-} = 1$ and k=1.5. The damage index evolution in the numerical simulation is represented in Figure 8.

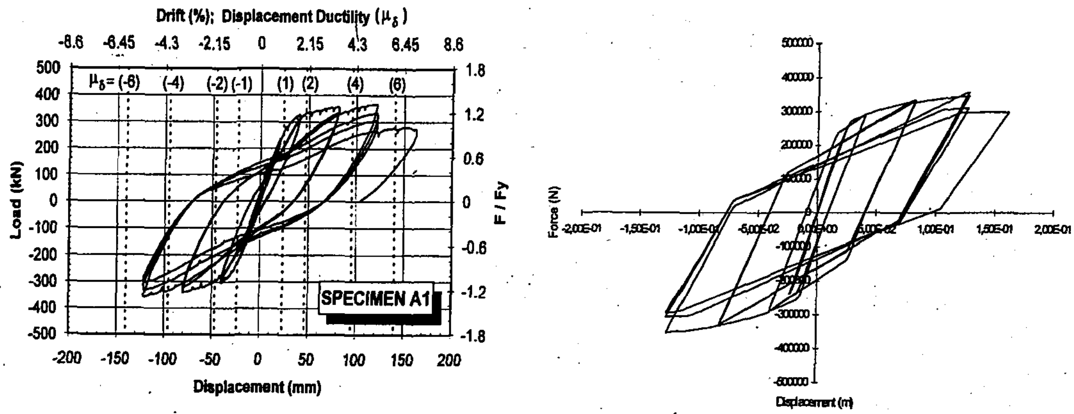

Figure 7. Experimental test (left) by Wehbe et al. (1996) and numerical simulation (right)

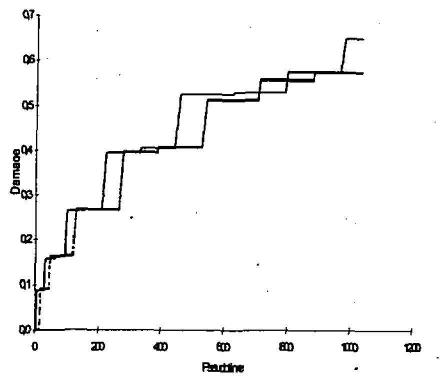

Figure 8. Damage evolution in the numerical simulation of the experimental test by Wehbe et al. (1996)

6 CONCLUSIONS

The results obtained are very hopeful. The model performs very well under monotonic and increased cyclic loading. The decrease of the strength due fatigue can be modeled using the proposed damage and plastic functions. A further study is needed in order to stablish a relationship among the k parameter and some geometric characteristics of the member as the longitudinal reinforcement ratio or the transverse reinforcement ratio. This parameter can also be affected by the normalized axial force. The pinching which the model is not able to represented will be introduced in the future, probably considering a non instantaneous closure of the cracks.

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